

# Consistent bilinear Wess-Zumino term for open AdS superstring

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## Abstract

We consider the open superstring action in  $\text{AdS}_5 \times \text{S}^5$  background with the bilinear Wess-Zumino term, which has been modified in such a way that a certain total derivative term is absent, and give the covariant description of supersymmetric D-branes. We show that the modification of bilinear Wess-Zumino term is necessary to describe correctly the 1/2-BPS D(-1)-brane in  $\text{AdS}_5 \times \text{S}^5$  background, while the classification of other supersymmetric D-branes does not depend on such modification.

Keywords : Wess-Zumino term, D-brane,  $\kappa$ -symmetry

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# 1 Introduction

The type IIB superstring in the  $\text{AdS}_5 \times \text{S}^5$  background is an important ingredient in the study of AdS/CFT correspondence [1, 2]. As is well known, it has been described in [3, 4] as a supersymmetric Green-Schwarz type sigma model based on the fact that the  $\text{AdS}_5 \times \text{S}^5$  background has the coset superspace structure. After the construction of its action, it has been observed in a similar construction in other AdS type background [5] that the symmetry superalgebra corresponding to the coset superspace has the  $\mathbf{Z}_4$ -automorphism. This has motivated an alternative description of type IIB superstring in the  $\text{AdS}_5 \times \text{S}^5$  background [6], which has provided a basis for the study of integrability in the superstring theory [7].<sup>1</sup>

The superficial difference between two descriptions is in the Wess-Zumino (WZ) term. In the conventional formulation by Metsaev and Tseytlin [3], the WZ term is given by

$$-2i \int_0^1 dt \int_{\Sigma} \widehat{L}^A \wedge \bar{\theta}^I \Gamma_{A\tau_3^{IJ}} \widehat{L}^J, \quad (1.1)$$

where  $\Sigma$  denotes the string worldsheet, and  $\widehat{L}^A$  ( $\widehat{L}^I$ ) is the vector (spinor) superfield or the Maurer-Cartan one-form superfield  $L^A$  ( $L^I$ ) with the rescaling  $\theta \rightarrow t\theta$ , that is,  $\widehat{L}^A(X, \theta) = L^A(X, t\theta)$  ( $\widehat{L}^I(X, \theta) = L^I(X, t\theta)$ ). (All the detailed expressions for the superfields including our notation and convention is given in the appendix.) From (1.1), we see that there is an integration in terms of an auxiliary parameter  $t$ . Contrary to this, the alternative formulation of [6] states that the WZ term does not involve such auxiliary integration and furthermore is manifestly bilinear with respect to the superfield, which is written as

$$\int_{\Sigma} \bar{L}^I \wedge \Gamma_* \tau_1^{IJ} L^J \quad (1.2)$$

in the 32 component notation.<sup>2</sup> It is obvious that the WZ term of (1.2) is simpler than that of (1.1) and hence seems to be advantageous in the study of superstring theory. Actually, it has been demonstrated that the bilinear WZ term is more practical to explore the algebraic or the dynamical aspect of type IIB superstring in the  $\text{AdS}_5 \times \text{S}^5$  background [9, 10, 11]. We would like to note that the same kind of bilinear WZ term arises also from the study of AdS/CFT correspondence for the non-critical strings[12].

One peculiar point in the structure of (1.2) is that there is a total derivative term,  $d\bar{\theta}^I \wedge \Gamma_* \tau_1^{IJ} d\theta^J$ , which is given as the leading order term when we expand the integrand in

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<sup>1</sup>For a comprehensive review on the development based on the alternative formulation of type IIB superstring in  $\text{AdS}_5 \times \text{S}^5$ , see [8] for example.

<sup>2</sup>By using  $\Gamma_* \equiv i\Gamma_{01234}$  of (A.6), and the ‘5+5’ way [3] of splitting the Dirac gamma matrices,  $\Gamma^a = \gamma^a \times 1 \times \sigma_1$ ,  $\Gamma^{a'} = 1 \times \gamma^{a'} \times \sigma_2$ , ( $a = 0, \dots, 4$ ,  $a' = 5, \dots, 9$ ) where  $\sigma_k$  are Pauli matrices, the integrand reduces to  $\bar{L}^I \wedge \tau_1^{IJ} L^J$ , which is the usual form considered in the literature.

terms of the fermionic coordinate  $\theta$  by using the expression of  $L^I$  given in (A.4). It has been pointed out in [13] that such a term should be subtracted from the WZ term to have the correct charge for the massive string excitations<sup>3</sup> and thus the bilinear WZ term of (1.2) should be modified as

$$S_{WZ} = \int_{\Sigma} (\bar{L}^I \wedge \Gamma_* \tau_1^{IJ} L^J - d\bar{\theta}^I \wedge \Gamma_* \tau_1^{IJ} d\theta^J). \quad (1.3)$$

The modification (1.3) has been proposed to deal with the problem in the closed string case. Now, one may be interested in the open string case and ask what the effect of the subtraction of the total derivative term is. In this paper, we address this question by considering the open string description of D-branes in the covariant setting. Firstly, following the prescription of [14], we consider the  $\kappa$ -symmetry variation of the superstring action. In order to make the action to be  $\kappa$ -symmetric, we impose suitable open string boundary conditions on the worldsheet boundary. In this way, we give the covariant description of D-branes and classify the supersymmetric 1/2-BPS D-branes in the  $\text{AdS}_5 \times \text{S}^5$  background. We then compare our result with that obtained in different contexts [15, 16]. As we will see, the comparison shows that it is necessary for the bilinear WZ term (1.2) to be modified as (1.3) for the full correct classification of 1/2-BPS D-branes. More precisely, the modified bilinear WZ term (1.3) should be used for the description of 1/2-BPS D(-1)-brane, that is, D-instanton. On the other hand, for other  $Dp$ -branes with  $p \geq 1$ , we do not need to care about the presence of the total derivative term.

This paper is organized as follows. The covariant open string description of 1/2-BPS D-branes is given in the next section. In Sec. 3, it is shown that the result of Sec. 2 is valid at full orders in the fermionic coordinate  $\theta$ . The final section is devoted to our conclusion. In Appendix, we give the expressions for the superfields together with the notations and conventions.

## 2 Covariant description of D-branes

In the original proposal for the covariant description of D-branes by using the Green-Schwarz open superstring action [17], an arbitrary variation of the action is considered and suitable open string boundary conditions for making the action invariant under the variation are investigated. However, as noted in [14], the very  $\kappa$ -symmetry variation, not arbitrary one, is enough at least for the description of supersymmetric D-branes, because the  $\kappa$ -symmetry

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<sup>3</sup>The charge corresponds to the string winding number in the flat space limit [13]. So, unless any direction is somehow compactified, it seems that the physics does not depend on the total derivative term.

leads to the matching of dynamical degrees of freedom for bosons and fermions on the worldsheet and hence ensures the object described by the open string supersymmetric.<sup>4</sup> In this section, we investigate the open string boundary conditions under which the superstring action with the modified bilinear WZ term of (1.3) is  $\kappa$ -symmetric.

The  $\kappa$ -symmetry transformation rules in superspace are given by

$$\delta_\kappa Z^M L_M^A = 0, \quad \delta_\kappa Z^M L_M^I = \eta^I, \quad \eta^I \equiv (\delta^{IJ} + \tau_3^{IJ} \Gamma) \kappa^J, \quad (2.1)$$

where  $\kappa^I$  is the  $\kappa$ -symmetry transformation parameter and  $\Gamma$  is basically the pullback of  $\Gamma_{AB}$  onto the string worldsheet with the properties,  $\Gamma^2 = 1$  and  $\text{Tr} \Gamma = 0$ , whose detailed expression is not needed here. Since the bulk part of the superstring action is  $\kappa$ -symmetric by construction, what we have under the  $\kappa$ -symmetry variation are the boundary contributions. It should be noted here that, as shown in [14], the kinetic part of the superstring action does not give any boundary contribution due to  $\delta_\kappa Z^M L_M^A = 0$  in (2.1). Thus we can focus only on the WZ term rather than the full superstring action.

The WZ term has an expansion in terms of the fermionic coordinate  $\theta$  up to the order of  $\theta^{32}$ . Although it is so, we will consider the expansion only up to quartic order in  $\theta$  in this section. As we will see in the next section, all the nontrivial information for the description of D-branes is obtained already from the terms in such restricted expansion. Then the WZ action (1.3) expanded up to the desired order is written as

$$S_{WZ} = S^0 + S^{\text{spin}} + S^{\mathcal{M}^2} + \dots, \quad (2.2)$$

where the dots represent the higher order terms and we have divided the terms of our interest into three parts, that is,  $\mathcal{M}^2$  dependent part  $S^{\mathcal{M}^2}$  (see Appendix for the definition of  $\mathcal{M}^2$ .), the spin connection dependent part  $S^{\text{spin}}$ , and the part  $S^0$  containing the remaining terms. These three parts have the following expressions.

$$\begin{aligned} S^0 &= \int_\Sigma \left( ie^A \wedge \bar{\theta}^I \Gamma_A \tau_3^{IJ} d\theta^J + \frac{1}{4} e^A \wedge e^B \bar{\theta}^I \Gamma_A \Gamma_* \Gamma_B \tau_1^{IJ} \theta^J \right), \\ S^{\text{spin}} &= - \int_\Sigma \omega^{AB} \wedge \left( \frac{1}{2} \bar{\theta}^I \Gamma_{AB} \Gamma_* \tau_1^{IJ} d\theta^J + \frac{1}{4^2} \omega^{CD} \bar{\theta}^I \Gamma_{AB} \Gamma_* \Gamma_{CD} \tau_1^{IJ} \theta^J + \frac{i}{4} e^C \bar{\theta}^I \Gamma_{ABC} \tau_3^{IJ} \theta^J \right), \\ S^{\mathcal{M}^2} &= \frac{1}{3} \int_\Sigma \overline{D\theta^I} \wedge \Gamma_* \tau_1^{IJ} (\mathcal{M}^2)^{IJ} D\theta^J. \end{aligned} \quad (2.3)$$

For the variation of these parts, it is now convenient to express the variation  $\delta_\kappa X^\mu$  in terms of  $\delta_\kappa \theta^I$  by using the transformation rule  $\delta_\kappa Z^M L_M^A = 0$  of (2.1) as follows:

$$\delta_\kappa X^\mu = -i \bar{\theta}^I \Gamma^\mu \delta_\kappa \theta^I + \mathcal{O}(\theta^4), \quad (2.4)$$

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<sup>4</sup>The suggestion using the  $\kappa$ -symmetry [14] has been demonstrated in the type IIB pp-wave background. In subsequent works, it has been successfully applied to other string theory backgrounds [16, 18].

where  $\mathcal{O}(\theta^4)$  leads to the terms of higher order than quartic one in the resulting variation of the WZ term and thus is not of our concern here. By utilizing this, we first consider the boundary contributions from the  $\kappa$ -symmetry variation of  $S^0$  found as

$$\delta_\kappa S^0 = \int_{\partial\Sigma} \left[ -idX^\mu e_\mu^A (\bar{\theta}^I \Gamma_A \tau_3^{IJ} \delta_\kappa \theta^J) + (\bar{\theta}^I \Gamma_A \tau_3^{IJ} d\theta^J) (\bar{\theta}^K \Gamma^A \delta_\kappa \theta^K) - \frac{i}{2} dX^\mu e_\mu^B (\bar{\theta}^I \Gamma_A \Gamma_* \Gamma_B \tau_1^{IJ} \theta^J) (\bar{\theta}^K \Gamma^A \delta_\kappa \theta^K) \right], \quad (2.5)$$

where  $\partial\Sigma$  represents the boundary of open string worldsheet. We have three non-vanishing terms on the right hand side. In order to have the  $\kappa$  invariance, they should vanish under a suitable set of open string boundary conditions. As for the first term, because

$$dX^A \equiv dX^\mu e_\mu^A = 0 \quad (A \in D), \quad (2.6)$$

where  $A \in D$  ( $N$ ) means that  $A$  is a direction of Dirichlet (Neumann) boundary condition,

$$\bar{\theta}^I \Gamma_A \tau_3^{IJ} \delta_\kappa \theta^J \quad (2.7)$$

should vanish for  $A \in N$ . For satisfying this, we impose the following 1/2-BPS boundary condition

$$\theta^I = P^{IJ} \theta^J \quad (2.8)$$

with

$$P^{IJ} = \begin{cases} s \Gamma^{A_1 \dots A_{p+1}} \tau_1^{IJ} & (p = 1 \bmod 4) \\ s \Gamma^{A_1 \dots A_{p+1}} \epsilon^{IJ} & (p = 3 \bmod 4) \end{cases}, \quad (2.9)$$

where all the indices  $A_1, \dots, A_{p+1}$  are those for Neumann directions and

$$s = \begin{cases} 1 & \text{for } X^0 \in N \\ i & \text{for } X^0 \in D \end{cases}. \quad (2.10)$$

We note that  $p$  should be odd because  $\theta^1$  and  $\theta^2$  have the same chirality and, for any odd  $p$ ,

$$P^{IJ} P^{JK} = \delta^{IK}, \quad \bar{\theta}^I = -\bar{\theta}^J P^{JI}. \quad (2.11)$$

Without much difficulty, we can now check that the boundary condition (2.8) makes the term of (2.7) vanish explicitly, that is,  $\bar{\theta}^I \Gamma_A \tau_3^{IJ} \delta_\kappa \theta^J = 0$  for  $A \in N$ . In turn, this result for the first term of (2.5) immediately leads us to have the vanishing condition for the second term as

$$\bar{\theta}^I \Gamma^A \delta_\kappa \theta^I = 0 \quad (A \in D). \quad (2.12)$$

The boundary condition (2.8) can be imposed again to show that this is indeed the case. So, up to this point, all odd  $p$ , that is,  $Dp$ -branes with  $p = -1, 1, 3, 5, 7, 9$  are possible.

The situation changes at the third term of (2.5). From Eqs. (2.6) and (2.12), we see that the vanishing condition of the term is

$$\bar{\theta}^I \Gamma_A \Gamma_* \Gamma_B \tau_1^{IJ} \theta^J = 0 \quad (A, B \in N). \quad (2.13)$$

Due to the presence of  $\Gamma_*$ , there are restrictions in the number of Neumann directions in  $\text{AdS}_5$  or  $S^5$  for satisfying this condition. Let us denote  $n$  ( $n'$ ) as the number of Neumann directions among  $0, \dots, 4$  ( $5, \dots, 9$ ). Then we have the relation,

$$n + n' = p + 1, \quad (2.14)$$

which means that both of  $n$  and  $n'$  are even or odd because  $p+1$  is even. Simple calculation shows that the condition (2.13) is satisfied for the following cases:

$$\begin{aligned} n, n' : \text{even} & \quad (p = 1 \bmod 4) \\ n, n' : \text{odd} & \quad (p = 3 \bmod 4). \end{aligned} \quad (2.15)$$

This gives us the information about the directions to which a 1/2-BPS D-brane can extend and shows that D9-brane is not 1/2-BPS. We would like to note here that the D(-1)-brane, that is, D-instanton is excluded in this restriction, since  $n = n' = 0$  is not allowed for  $p = 3 \bmod 4$ . However, the third term vanishes even for the D-instanton case, because  $dX^A = 0$  for all  $A$  for the D-instanton boundary condition. Thus, the D-instanton is also one of the 1/2-BPS D-branes.

We now turn to the boundary contributions from the  $\kappa$ -symmetry variation of  $S^{\text{spin}}$ , which are found as

$$\begin{aligned} \delta_\kappa S^{\text{spin}} = \frac{1}{2} \int_{\partial\Sigma} & \left[ (\bar{\theta}^I \Gamma_{AB} \Gamma_* \tau_1^{IJ} \delta_\kappa \theta^J) dX^\mu + \frac{1}{2} (\bar{\theta}^I \Gamma_C \Gamma_{AB} \tau_3^{IJ} \theta^J) (\bar{\theta}^K \Gamma^C \delta_\kappa \theta^K) dX^\mu \right. \\ & \left. + i (\bar{\theta}^I \Gamma_{AB} \Gamma_* \tau_1^{IJ} D\theta^J) (\bar{\theta}^K \Gamma^\mu \delta_\kappa \theta^K) \right] \omega_\mu^{AB}. \end{aligned} \quad (2.16)$$

As for the first two terms on the right hand side, by repeating the same procedure applied to  $\delta_\kappa S^0$  of (2.5), the boundary condition (2.8) with the restriction (2.15) leads us to have some spinor bilinears which vanish at the boundary,

$$\begin{aligned} \bar{\theta}^I \Gamma_{AB} \Gamma_* \tau_1^{IJ} \delta_\kappa \theta^J &= 0 \quad (A, B \in N(D)), \\ \bar{\theta}^I \Gamma_C \Gamma_{AB} \tau_3^{IJ} \theta^J &= 0 \quad (A, B \in N(D), C \in N). \end{aligned} \quad (2.17)$$

Although these eliminate the boundary contributions with the corresponding index structure, other contributions do not vanish. By the way, one common property of those surviving contributions is that they are proportional to the spin connection  $\omega^{AB}$  with  $A \in N$  and

$B \in D$  (or  $A \in D$  and  $B \in N$ ). From the expression of the spin connection (A.7), we see that those contributions vanish if the Dirichlet directions are set to zero. This means that a given D-brane is 1/2-BPS if it is placed at the coordinate origin in its transverse directions.

For the consideration of the third term on the right hand side of (2.16), the following relation at the boundary is useful.

$$(DP\theta)^I = (PD\theta)^I - \omega^{AB}\Gamma_{AB}P^{IJ}\theta^J, \quad (2.18)$$

where  $A \in N$  and  $B \in D$ . By utilizing this, we can show that all the non-vanishing boundary contributions are proportional to  $\omega^{AB}$  with  $A \in N$  and  $B \in D$  (or  $A \in D$  and  $B \in N$ ) like the case of first two terms of (2.16), and hence vanish at the origin of the Dirichlet directions and we have the  $\kappa$ -invariance. As a remark, we would like to note that the D-instanton is actually exceptional because the whole boundary contributions from  $\delta_\kappa S^{\text{spin}}$  vanish basically because of (2.6) and (2.12). Thus, the D-instanton is 1/2-BPS in every position.

Finally, there are boundary contributions from  $\delta_\kappa S^{\mathcal{M}^2}$  which are obtained as

$$\begin{aligned} \delta_\kappa S^{\mathcal{M}^2} = \frac{1}{6} \int_{\partial\Sigma} & \left[ 2(\bar{\theta}^I \Gamma_A \delta_\kappa \theta^I)(\bar{\theta}^J \Gamma^A \tau_3^{JK} D\theta^K) - 2(\bar{\theta}^I \Gamma_A D\theta^I)(\bar{\theta}^J \Gamma^A \tau_3^{JK} \delta_\kappa \theta^K) \right. \\ & + (\bar{\theta}^I \Gamma_{ab} \Gamma_* \epsilon^{IJ} \delta_\kappa \theta^J)(\bar{\theta}^K \Gamma^{ab} \Gamma_* \tau_1^{KL} D\theta^L) \\ & - (\bar{\theta}^I \Gamma_{ab} \Gamma_* \epsilon^{IJ} D\theta^J)(\bar{\theta}^K \Gamma^{ab} \Gamma_* \tau_1^{KL} \delta_\kappa \theta^L) \\ & - (\bar{\theta}^I \Gamma_{a'b'} \Gamma_*' \epsilon^{IJ} \delta_\kappa \theta^J)(\bar{\theta}^K \Gamma^{a'b'} \Gamma_* \tau_1^{KL} D\theta^L) \\ & \left. + (\bar{\theta}^I \Gamma_{a'b'} \Gamma_*' \epsilon^{IJ} D\theta^J)(\bar{\theta}^K \Gamma^{a'b'} \Gamma_* \tau_1^{KL} \delta_\kappa \theta^L) \right]. \end{aligned} \quad (2.19)$$

We have checked that the boundary contributions vanish without any additional condition. However, we do not give any detailed explanation, because the term containing  $\mathcal{M}^2$  as well as the terms of higher powers of  $\mathcal{M}^2$  will be dealt with all at once in the next section. So, we complete the investigation of the open string boundary condition for the  $\kappa$ -symmetry of the action expanded up to quartic order in  $\theta$ , and hence the classification of 1/2-BPS D-branes, which is summarized in the table 1. We note that our D-brane classification is in complete agreement with that from the probe analysis [15] and from the analysis using the WZ term constructed by Metsaev and Tseytlin (1.1) [16].

In our study, we have taken the modified bilinear WZ term (1.3) which does not include the total derivative term. Before going to the next section, we consider the problem as to whether the classification of 1/2-BPS D-branes is valid even if such total derivative term is included in the WZ term. Under the  $\kappa$ -symmetry variation, we have

$$\delta_\kappa \int_{\Sigma} d\bar{\theta}^I \wedge \Gamma_* \tau_1^{IJ} d\theta^J = 2 \int_{\partial\Sigma} \delta_\kappa \bar{\theta}^I \Gamma_* \tau_1^{IJ} d\theta^J, \quad (2.20)$$

$Dp$	$(n, n')$
D(-1)	(0, 0)
D1	(2, 0) (0, 2)
D3	(1, 3) (3, 1)
D5	(2, 4) (4, 2)
D7	(3, 5) (5, 3)
D9	absent

Table 1: 1/2-BPS D-branes in the  $\text{AdS}_5 \times S^5$  background.  $n$  ( $n'$ ) is the number of Neumann directions in  $\text{AdS}_5$  ( $S^5$ ).

where there are only boundary contributions because the variation of total derivative term does not give bulk contribution. It is not difficult to show that the boundary condition (2.8) with the restriction (2.15) makes the boundary contributions vanish and ensures the  $\kappa$ -invariance. On the other hand, we find that the boundary condition for the D-instanton, which is not in the criterion of (2.15), cannot eliminate the right hand side of (2.20). Thus, the inclusion of the total derivative term does not lead to the 1/2-BPS D-instanton while it does not affect the classification of 1/2-BPS  $Dp$ -branes with  $p \geq 1$ .<sup>5</sup> This means that we should use the modified bilinear WZ term (1.3) for the full correct classification of 1/2-BPS D-branes.

### 3 Validity at full orders in $\theta$

We have shown that the boundary condition (2.8) with the restriction (2.15) makes the boundary contributions from the  $\kappa$ -symmetry variation of the WZ term vanish up to the quartic order in  $\theta$ . In this section, we provide a proof that such boundary condition is sufficient for showing the boundary  $\kappa$ -symmetry of the WZ term even at higher orders in  $\theta$  without any extra boundary condition, and thus the classification of 1/2-BPS D-branes in the  $\text{AdS}_5 \times S^5$  background summarized in the table 1 is valid at full orders in  $\theta$ .

For the investigation of  $\kappa$ -symmetry at higher orders in  $\theta$ , it is not necessary to take the modified bilinear WZ term (1.3) since the subtracted total derivative term is the leading order one. Thus it is enough to consider the unmodified bilinear WZ term of (1.2). Then the boundary contribution from the  $\kappa$ -symmetry variation of this term is obtained as

$$2 \int_{\partial\Sigma} \bar{\eta}^I \Gamma_* \tau_1^{IJ} L^J, \quad (3.1)$$

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<sup>5</sup>Related to our result, it has been reported that the D-instanton is distinguished from other D-branes also in the construction of D-brane action in AdS space [19].



where we have used the  $\kappa$ -symmetry transformation rule (2.1). In order to proceed, we need to know the boundary condition of  $\eta^I$ . Since we can see that  $\delta_\kappa \theta^I = \eta^I + \mathcal{O}(\theta^2)$  from the transformation rule (2.1), it is natural to expect that the boundary condition of  $\eta^I$  is the same with that of  $\theta^I$ , that is,  $\eta^I = P^{IJ} \eta^J$  (or  $\bar{\eta}^I = -\bar{\eta}^J P^{JI}$ ). Although this seems a naive expectation, it has been shown rigorously in [20] that this is indeed the case.<sup>6</sup> If we now impose this boundary condition in (3.1) and carry out a bit of manipulation with the condition (2.15), then we have

$$-2 \int_{\partial\Sigma} \bar{\eta}^I \Gamma_* \tau_1^{IJ} P^{JK} L^K. \quad (3.2)$$

This expression tells us that the boundary contribution vanishes if the spinor superfield satisfies the condition

$$L^I = P^{IJ} L^J, \quad (3.3)$$

at the worldsheet boundary  $\partial\Sigma$ , because  $\bar{\eta}^I \Gamma_* \tau_1^{IJ} L^J = -\bar{\eta}^I \Gamma_* \tau_1^{IJ} P^{JK} L^K = -\bar{\eta}^I \Gamma_* \tau_1^{IJ} L^J$  means  $\bar{\eta}^I \Gamma_* \tau_1^{IJ} L^J = 0$ . In what follows, we will show that the spinor superfield indeed follows the boundary condition (3.3).

Let us first consider the boundary condition for  $Dp$ -branes with  $p \geq 1$ , leaving the discussion of the D-instanton case separately. In order to see the effect of imposing the boundary condition on the spinor superfield  $L^I$ , we focus on the term of the form  $\mathcal{M}^{2n} D\theta$ , which is the field dependent part of the summand in the series expression of  $L^I$  (A.4). For the elementary piece  $\mathcal{M}^2$ , it is not difficult to show that

$$(\mathcal{M}^2)^{IJ} = P^{IK} (\mathcal{M}^2)^{KL} P^{LJ} \quad (3.4)$$

by using the definition of  $\mathcal{M}^2$  given in (A.5) and the boundary condition (2.8) with (2.15). This means that  $(\mathcal{M}^{2n})^{IJ} = P^{IK} (\mathcal{M}^{2n})^{KL} P^{LJ}$  from the property of  $P^{IJ}$  (2.11) and in turn we get  $(\mathcal{M}^{2n})^{IJ} (D\theta)^J = P^{IJ} (\mathcal{M}^{2n})^{JK} (PD\theta)^K$  at the boundary. Now the question is whether or not the relation  $(D\theta)^I = (PD\theta)^I$  (or  $(DP\theta)^I = (PD\theta)^I$ ) holds at the boundary. As mentioned in (2.18) in the previous section, this relation does not hold generically and  $(DP\theta)^I$  differs from  $(PD\theta)^I$  by an amount of spin connection dependent term. However, what we are interested in here are the 1/2-BPS  $Dp$ -branes with  $p \geq 1$ , which should be located at the coordinate origin in the transverse directions. Because the spin connection vanishes at such position, we can set  $(DP\theta)^I = (PD\theta)^I$  at least for the description of 1/2-BPS D-branes. As a result of this, it turns out that the spinor superfield satisfies the

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<sup>6</sup>For the rigorous proof, consult the procedure of proving Eq. (3.4) of [20].

boundary condition (3.3). We would like to note that, in showing the boundary condition (3.3), we have not imposed any additional boundary condition other than that introduced in the previous section. Therefore we conclude that the classification of 1/2-BPS  $Dp$ -branes ( $p \geq 1$ ) in the previous section is valid even at higher orders in  $\theta$ .

We now turn to the D-instanton case. This is the special case in the sense that the sign in (3.2) is positive instead of negative basically because the D-instanton is not included in the criterion given in (2.15) and thus the previous argument for showing the vanishing of the boundary contribution from the  $\kappa$ -symmetry transformation is not applicable. Actually, this is the reason that the D-instanton should be treated separately.

Let us observe that  $D\theta^I = d\theta^I$  at the boundary for the D-instanton boundary condition because  $dX^A = 0$  for all  $A$ . By using this fact and the D-instanton boundary condition matrix  $P^{IJ} = i\epsilon^{IJ}$  from (2.9), we can easily show that  $(\mathcal{M}^2)^{IJ}D\theta^J = (\mathcal{M}^2)^{IJ}d\theta^J = 0$ . This immediately means that the spinor superfield is given by  $L^I = d\theta^I$  at the boundary, which simplifies the  $\kappa$ -symmetry transformation (2.1) related to  $L^I$  as  $\delta_\kappa\theta = \eta^I$ . Therefore, the boundary contribution (3.1) does not vanish and becomes  $2 \int_{\partial\Sigma} \delta_\kappa \bar{\theta}^I \Gamma_* \tau_1^{IJ} d\theta^J$ . However, this is the term exactly cancelled by the boundary contribution from the total derivative term in (1.3) which has been omitted in this section, and thus we have  $\kappa$ -symmetry as a whole.

In conclusion, since we do not have to introduce any additional boundary condition for showing the  $\kappa$ -symmetry invariance of the action even at higher orders in  $\theta$ , it is verified that the classification of 1/2-BPS D-branes in the previous section is valid at full orders in  $\theta$ .

## 4 Conclusion

We have taken the type IIB superstring action in the  $\text{AdS}_5 \times \text{S}^5$  background whose WZ term is of the bilinear form, and given a covariant open string description of 1/2-BPS D-branes in the background. Under the  $\kappa$ -symmetry transformation, some boundary contributions appear from the variation of the WZ term. A set of suitable open string boundary conditions for making them vanish has been investigated to have the full  $\kappa$ -symmetry and, as a result, the classification of possible 1/2-BPS D-branes has been obtained with its validity check at full orders in  $\theta$ . As it should be, our result agrees exactly with the previous classification [15, 16].

The important point in our study is that the bilinear WZ term should not have the total derivative term for the correct D-brane classification. Actually, the description of 1/2-BPS  $Dp$ -branes with  $p \geq 1$  is not sensitive for such term. However the total derivative term

should be absent for describing the 1/2 BPS D-instanton. Related to the present work, there has been an attempt to describe the D-branes in the  $\text{AdS}_5 \times S^5$  background from a different perspective, the integrability, which is based on the same action as ours but keeping the total derivative term [21]. One of the results was that D-instanton was not in the class of integrable boundary condition and excluded in the D-brane classification. Although our focus is not the integrability and thus the direct comparison of our result with that obtained in [21] may not be sensible, one may expect carefully that the absence of the integrable boundary condition for the supersymmetric D-instanton may be related to the presence of the total derivative term in the bilinear WZ term.

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## A Supergeometry of the $\text{AdS}_5 \times S^5$ background

The notation for the supercoordinate we use is

$$Z^M = (X^\mu, \theta^I), \quad (\text{A.1})$$

where the spinor index for the fermionic coordinate  $\theta$  has been suppressed,  $\mu$  is the ten dimensional curved space-time vector index, and  $I (= 1, 2)$  is introduced to distinguish the two same chirality spinors. As for the Lorentz frame or the tangent space, the vector index is denoted by

$$A = (a, a'), \quad a = 0, 1, 2, 3, 4, \quad a' = 5, \dots, 9, \quad (\text{A.2})$$

where  $a$  ( $a'$ ) corresponds to the tangent space of  $\text{AdS}_5$  ( $S^5$ ), and the metric  $\eta_{AB}$  follows the most plus sign convention as  $\eta_{AB} = \text{diag}(-, +, +, \dots, +)$ .

The matrices acting on the spinors indexed with  $I, J, \dots$  are denoted by

$$\tau_i^{IJ}, \quad (i = 1, 2, 3), \quad (\text{A.3})$$

which are the usual Pauli matrices.

The explicit expression for the vector (spinor) superfield or the Maurer-Cartan one-form superfield  $L^A = dZ^M L_M^A$  ( $L^I = dZ^M L_M^I$ ) is given by [3, 4]

$$\begin{aligned} L^A &= e^A + 2i \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta}^I \Gamma^A (\mathcal{M}^{2n})^{IJ} D\theta^J, \\ L^I &= \sum_{n=0}^{16} \frac{1}{(2n+1)!} (\mathcal{M}^{2n})^{IJ} D\theta^J, \end{aligned} \quad (\text{A.4})$$

where  $\mathcal{M}^2$  and the spinor covariant derivative  $D\theta^I$  are, in the 32 component notation,

$$\begin{aligned} (\mathcal{M}^2)^{IJ} &= -\epsilon^{IK} \Gamma_* \Gamma^A \theta^K \bar{\theta}^J \Gamma_A + \frac{1}{2} \epsilon^{JK} (\Gamma^{ab} \theta^I \bar{\theta}^K \Gamma_{ab} \Gamma_* - \Gamma^{a'b'} \theta^I \bar{\theta}^K \Gamma_{a'b'} \Gamma_*), \\ D\theta^I &= \left( d + \frac{1}{4} \omega^{AB} \Gamma_{AB} \right) \theta^I - \frac{i}{2} \epsilon^{IJ} e^A \Gamma_* \Gamma_A \theta^J. \end{aligned} \quad (\text{A.5})$$

with the convention  $\epsilon^{12} = 1$  for the antisymmetric tensor  $\epsilon^{IJ}$  ( $= i\tau_2^{IJ}$ ). Some definitions of gamma matrix products and their properties are as follows.<sup>7</sup>

$$\begin{aligned} \Gamma_* &\equiv i\Gamma_{01234}, \quad \Gamma'_* \equiv i\Gamma_{56789}, \quad \Gamma_*^2 = 1, \quad \Gamma'^2_* = -1, \\ \Gamma^{11} &= \Gamma^{01\dots 9} = \Gamma_* \Gamma'_*, \quad (\Gamma^{11})^2 = 1. \end{aligned} \quad (\text{A.6})$$

The zehnbein and the corresponding spin connection for the  $\text{AdS}_5 \times \text{S}^5$  are given by [13]

$$\begin{aligned} e^a &= dX^a + \left( \frac{\sinh X}{X} - 1 \right) dX^b Y_b^a, \quad e^{a'} = dX^{a'} + \left( \frac{\sinh X'}{X'} - 1 \right) dX^{b'} Y_{b'}^{a'}, \\ \omega^{ab} &= \frac{1}{2} \left( \frac{\sinh(X/2)}{X/2} \right)^2 dX^{[a} X^{b]}, \quad \omega^{a'b'} = -\frac{1}{2} \left( \frac{\sinh(X'/2)}{X'/2} \right)^2 dX^{[a'} X^{b']}, \end{aligned} \quad (\text{A.7})$$

where

$$\begin{aligned} X &= \sqrt{X^a X_a}, \quad X' = \sqrt{X^{a'} X_{a'}}, \\ Y_a^b &= \delta_a^b - \frac{X_a X^b}{X^2}, \quad Y_{a'}^{b'} = \delta_{a'}^{b'} - \frac{X_{a'} X^{b'}}{X'^2}. \end{aligned} \quad (\text{A.8})$$

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<sup>7</sup>We follow the notation adopted in [22].

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